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TWO-STAGE CHAIN SAMPLING INSPECTION PLANS WITH DIFFERENT SAMPLE SIZES IN THE TWO STAGES

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INTRODUCTION

In previous reports[1,2,3,4] a generalized family of two-stage chain sampling inspection plans was presented. Conditions for their use were also discussed. This report contains a further generalization of these plans with particular attention to the specification of the sample sizes used in the different stages of the criteria. Previous plans specify the use of the same sample size in both stages. The plans presented here call for the use of different sample sizes in the two stages.

While this modification results in a variable amount of inspection, it has certain advantages. In fact, it seems logical to require a larger sample during initial start-up and following rejected lots, for, in practice, it often happens that defective lots occur in bunches. Under these conditions if a defective lot is found, as suggested by a rejection, the succeeding lots warrant closer inspection. In principle, the inspection procedures for both the military standard plans (MIL-STD-105D[5]), and the continuous sampling plans (such as CSP-1[6]) also incorporate intensified inspections during periods of possibly excessive defectiveness. MIL-STD-105D provides for "Tightened Inspection" when lot rejections are too closely spaced, while CSP-1 requires

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100% inspection (to the extent of clearing i consecutive units) following the occurrence of a defective during sampling.

Comparisons of the new plans with those having the same parameters but a single sample size indicate that improved discrimination is achieved by this two-sample-size procedure. The evaluation of the new plans is carried out using the framework of the theory described in a previous report.[3]

GENERAL PLAN AND OPERATING PROCEDURE

The general plan and operating procedure are the same as described in the above-mentioned previous reports. The parameters designating the number of samples over which cumulation takes place in the different stages i.e., k_1 and k_2, and the parameters designating the allowable number of defectives in the associated cumulative results i.e., C_1 and C_2, remain the same. The parameter designating the sample size i.e., n, is now defined separately for each of the two stages--namely n_1 and n_2.

For purposes of illustration and completeness, a modified schematic of the operating procedure given in the previous reports is shown in Fig. 1.

The following designations are used in the schematic to describe the operation of the sampling procedure:

d_i--the number of defectives in the ith sample.

D_i--the cumulative number of defectives at the ith sample, with cumulation performed as shown.

Here the definition of D_i differs from that used in the earlier report.[1]

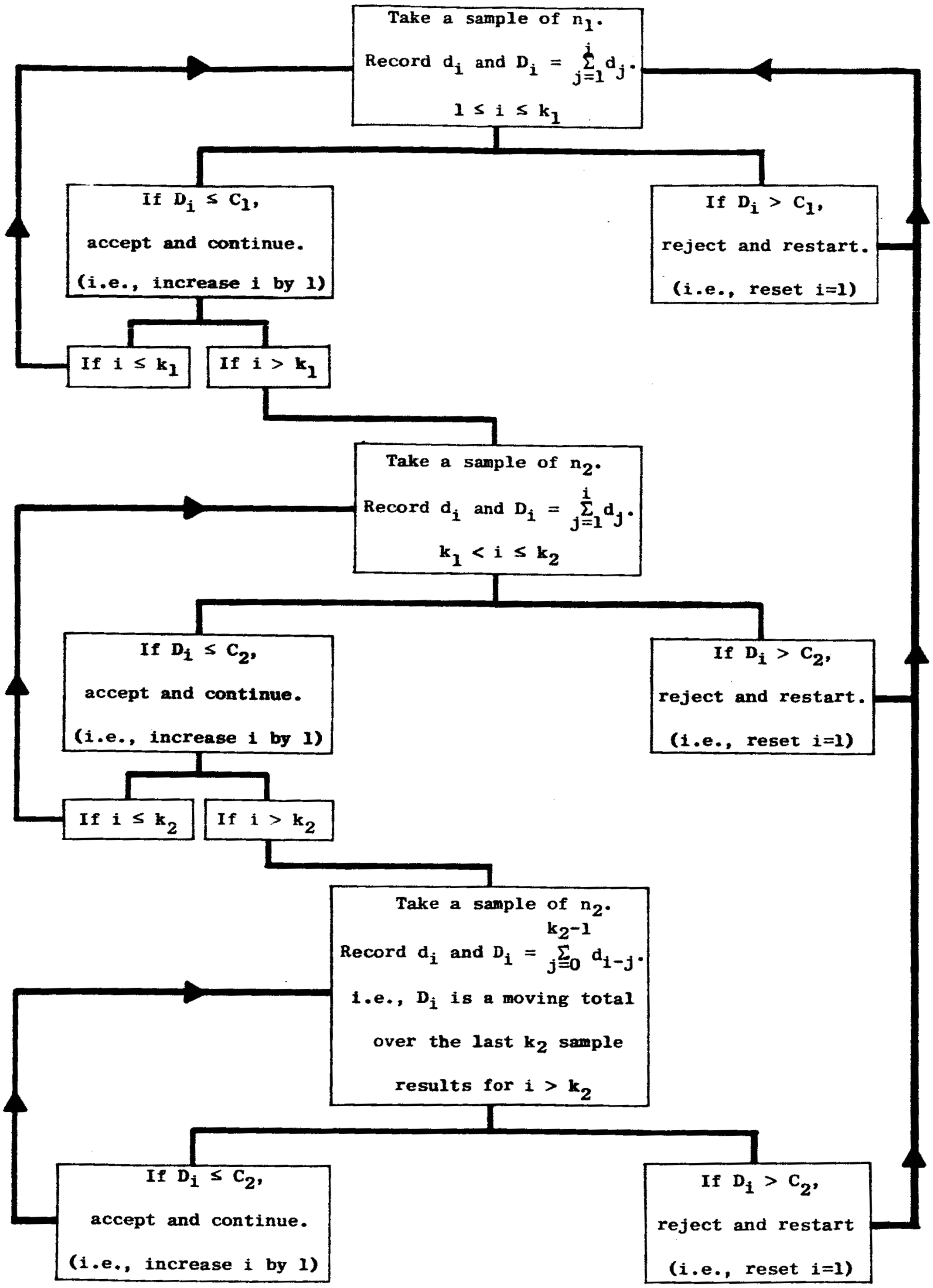

Fig. 1 Flow Chart of Operations, Two-Stage Chain Sampling Plan with Different n, i.e. n_1 and n_2

DESIGNATION OF PLANS

It is of course possible to assign a large number of different values to the six parameters which make up a plan. To facilitate discussion the general system of designation defined before will again be used, with ChSP(n_1,n_2)-C_1,C_2 designating the "Chain Sampling Plan with Different n, i.e. n_1 and n_2" with cumulative-result acceptance numbers C_1 and C_2:

Parameters						Designation
n_1,	n_2;	k_1,	k_2;	C_1,	C_2	
n_1,	n_2;	k_1,	k_2;	0,	1	ChSP(n_1,n_2)-0,1
n_1,	n_2;	k_1,	k_2;	0,	2	ChSP(n_1,n_2)-0,2
n_1,	n_2;	k_1,	k_2;	1,	2	ChSP(n_1,n_2)-1,2
n_1,	n_2;	k_1,	k_2;	0,	3	ChSP(n_1,n_2)-0,3
n_1,	n_2;	k_1,	k_2;	1,	3	ChSP(n_1,n_2)-1,3
			etc.			etc.

In referring to a specific plan, the values of the parameters in the above order will be enclosed in parentheses e.g., (20, 10; 1, 2; 0, 2) designates the ChSP(n_1,n_2)-0,2 plan having parameters: $n_1 = 20$, $n_2 = 10$; $k_1 = 1$, $k_2 = 2$; $C_1 = 0$, $C_2 = 2$. On occasion it will be convenient to abridge the notation to the four basic parameters, when n_1 and n_2 are clearly implied.

EVALUATING OPERATING CHARACTERISTICS

The operating characteristics of the present procedure are readily evaluated using the Markov chains developed for evaluating the ChSP procedures of the previous reports. The only necessary modification is to make each transition probability a function of the parameters n_1 and n_2, depending on whether the transition involves the first or second stage of the criteria. In general, the transition probabilities from those states not involving an "R" (denoting a rejection) will always be functions of n_2 only, since these states are reached only while in the second stage of the criteria. The transition probabilities from state "R" will always be functions of n_1 only, since a rejection requires a return to the first stage of the criteria. The transition probabilities from other states involving an R, e.g., (R0, R1, R2, R00, R01, R10, R02, R02, R000, etc.) will be functions of n_1 or n_2 depending on the parameter k_1. When the number of sample outcomes contained in a state involving a rejection (R) are equal to or greater than $k_1 + 1$, transitions from such states will be functions of n_2. But when the number of sample outcomes contained in a state involving a rejection (R) are less than $k_1 + 1$, transitions from these states will be functions of n_1. To illustrate the situation for states involving an R other than the state "R" itself, the following examples are given.

Example 1

Let $k_1 = 1$, $k_2 = 4$; $C_1 = 0$, $C_2 = 2$.

States involving an R other than the state "R" itself are:

$$\left.\begin{matrix} R00 \\ R01 \\ R02 \end{matrix}\right\} \text{number of sample outcomes contained in each state equal 3,}$$

R0 - number of sample outcomes contained in this state equal 2.

The number of sample outcomes contained in each of these states $\geq k_1 + 1 = 2$. Hence transitions from these states are functions of n_2, since passage to the second stage of the criteria takes place with the occurrence of zero defectives following a rejection (i.e., $k_1 = 1$). The possible transitions from the above states in accordance with the given acceptance criteria are as follows:

$p_{R00,000}$ = Probability of zero defectives in n_2

$p_{R00,001}$ = Probability of one defective in n_2

$p_{R00,002}$ = Probability of two defectives in n_2

$p_{R00,R}$ = Probability of three or more defectives in n_2

$p_{R01,010}$ = Probability of zero defectives in n_2

$p_{R01,011}$ = Probability of zero defectives in n_2

$p_{R01,R}$ = Probability of two or more defectives in n_2

$p_{R02,020}$ = Probability of zero defectives in n_2

$p_{R02,R}$ = Probability of one or more defectives in n_2

$p_{R0,R00}$ = Probability of zero defectives in n_2

$p_{R0,R01}$ = Probability of one defective in n_2

$p_{R0,R02}$ = Probability of two defectives in n_2

$p_{R0,R}$ = Probability of three or more defectives in n_2

Now note that for $k_1 = 2$, $k_2 = 4$; $C_1 = 0$, $C_2 = 2$, only two of the above states are admissible--namely R00 and R0, i.e., R01 and R02 exceed the $C_1 = 0$ criterion for $k_1 = 2$ samples following a rejection.

For the first of the admissible states involving a rejection i.e., R00, the number of sample outcomes contained in the state = $k_1 + 1 = 3$.

Hence transitions from this state are functions of n_2 and are the same as in the above example.

However, the number of sample outcomes contained in the state R0 $< k_1 + 1 = 3$. Hence transitions from this state are functions of n_1 since two (i.e., k_1) acceptable sample outcomes must follow a rejection before passing to the second stage of the criteria.

The possible transitions from this state in accordance with the given acceptance criteria are as follows:

$p_{R0,R00}$ = Probability of zero defectives in n_1.

$p_{R0,R}$ = Probability of one or more defectives in n_1.

Example 2

Let $k_1 = 3$, $k_2 = 4$; $C_1 = 0$, $C_2 = 2$.

For this case, states R00 and R0 are again admissible. However, the number of sample outcomes contained in both states R00 and R0 $< k_1 + 1 = 4$. Hence transitions from both states are functions of n_1.

The possible transitions are as follows:

$p_{R00,000}$ = Probability of zero defectives in n_1

$p_{R00,R}$ = Probability of one or more defectives in n_1

$p_{R0,R00}$ = Probability of zero defectives in n_1

$p_{R0,R}$ = Probability of one or more defectives in n_1

Previously (in the earlier reports) the transition probabilities, p_{ij}, were labeled: P_d, $d = 0, 1, \ldots, C_2$ and defined accordingly,

P_d = Probability of d defectives in a sample of n

For this report the transition probabilities, p_{ij} are labeled:

P_{d,n_s}, $d = 0, 1, \ldots, C_2$; $s = 1, 2$ and are defined as,

P_{d,n_s} = Probability of d defectives in a sample of n_s.

Making use of this notation, the transition probability matrices for a number of Markov cahins are given below. These are Markov chains for a selected group of the sets of parameters which have been evaluated. The following nine plans are illustrated:

Plan	k_1,	k_2;	C_1,	C_2
(1)	1,	2;	0,	1
(2)	1,	3;	0,	1
(3)	2,	3;	0,	1
(4)	1,	2;	0,	2
(5)	1,	3;	0,	2
(6)	2,	3;	1,	2
(7)	1,	5;	0,	2
(8)	1,	2;	0,	3
(9)	2,	3;	0,	3

Algebraic solutions for $\mathcal{P}a$, the proportion of lots expected to be accepted, are given for plans (1), (2), (3), (4), and (8) by solving the Markov chains for the limiting probability of the rejection, "R", state.

Plan (1): 1,2; 0,1

State at ith trial

State at (i-1)st trial		0	1	R
	0	P_{0,n_2}	P_{1,n_2}	$1 - \sum_{i=0}^{1} P_{i,n_2}$
	1	P_{0,n_2}	.	$1 - P_{0,n_2}$
	R	P_{0,n_1}	.	$1 - P_{0,n_1}$

Fig. 2. Markov Chain for ChSP(n_1,n_2)-0,1 Plan: (k_1,k_2; C_1,C_2 = 1,2; 0,1)

Proceeding as in previous reports[3,4], the solution of the limiting probability of state "R" from which $\mathcal{P}a$ is obtained is as follows:

$$\mathcal{P}_0 = P_{0,n_2}\mathcal{P}_0 + P_{0,n_2}\mathcal{P}_1 + P_{0,n_1}\mathcal{P}_R \tag{1}$$

$$\mathcal{P}_1 = P_{1,n_2}\mathcal{P}_0 \tag{2}$$

$$\mathcal{P}_0 + \mathcal{P}_1 + \mathcal{P}_R = 1 \tag{3}$$

Using (2) in (1) produces the following equations:

$$\rho_0 = \frac{P_{0,n_1}\rho_R}{1 - P_{0,n_2} - P_{0,n_2}P_{1,n_2}} \tag{4}$$

$$\rho_1 = \frac{P_{0,n_1}P_{1,n_2}\rho_R}{1 - P_{0,n_2} - P_{0,n_2}P_{1,n_2}} \tag{5}$$

Using (4) and (5) in (3) gives the result for ρ_R

$$\rho_R = \frac{1 - P_{0,n_2} - P_{0,n_2}P_{1,n_2}}{1 + P_{0,n_1} - P_{0,n_2} + P_{0,n_1}P_{1,n_2} - P_{0,n_2}P_{1,n_2}} \tag{6}$$

$$\rho a = 1-\rho_R = \frac{P_{0,n_1} + P_{0,n_1}P_{1,n_2}}{1 + P_{0,n_1} - P_{0,n_2} + P_{0,n_1}P_{1,n_2} - P_{0,n_2}P_{1,n_2}} \tag{7}$$

It is of interest to check the agreement of this result with the special case of $n_1 = n_2 = n$, for which the parameters being considered are those of a ChSP-0,1 plan[1] or more specifically, the ChSP-1 plan[7] with i = 1.

With $P_{0,n_1} = P_{0,n_2} = P_0$, and $P_{1,n_2} = P_1$, (7) reduces to,

$\rho a = P_0 + P_0P_1$, which is the expected result.

<u>Plan (2): 1,3; 0,1</u>

State at ith trial

State at (i-1)st trial		00	01	10	R0	R
	00	P_{0,n_2}	P_{1,n_2}	·	·	$1 - \sum_{i=0}^{1} P_{i,n_2}$
	01	·	·	P_{0,n_2}	·	$1 - P_{0,n_2}$
	10	P_{0,n_2}	·	·	·	$1 - P_{0,n_2}$
	R0	P_{0,n_2}	P_{1,n_2}	·	·	$1 - \sum_{i=0}^{1} P_{i,n_2}$
	R	·	·	·	P_{0,n_1}	$1 - P_{0,n_1}$

Fig. 3. Markov Chain for ChSP(n_1,n_2)-0,1 Plan: (k_1,k_2; C_1,C_2 = 1,3; 0,1)

$$\rho_{00} = P_{0,n_2}\rho_{00} + P_{0,n_2}\rho_{10} + P_{0,n_2}\rho_{R0} \tag{8}$$

$$\rho_{01} = P_{1,n_2}\rho_{00} + P_{1,n_2}\rho_{R0} \tag{9}$$

$$\rho_{10} = P_{0,n_2}\rho_{01} \tag{10}$$

$$\rho_{R0} = P_{0,n_1}\rho_{R} \tag{11}$$

$$\rho_{00} + \rho_{01} + \rho_{10} + \rho_{R0} + \rho_{R} = 1 \tag{12}$$

Combining equations as before,

$$\rho_R = \frac{1 - P_{0,n_2} - P_{0,n_2}^2 P_{1,n_2}}{1-P_{0,n_2}-P_{0,n_2}P_{1,n_2}+P_{0,n_1}+P_{0,n_1}P_{1,n_2}+P_{0,n_1}P_{0,n_2}P_{1,n_2}} \tag{13}$$

$$\rho_a = \frac{P_{0,n_1}+P_{0,n_1}P_{1,n_2}+P_{0,n_1}P_{0,n_2}P_{1,n_2}}{1-P_{0,n_2}-P_{0,n_2}^2 P_{1,n_2}+P_{0,n_1}+P_{0,n_1}P_{1,n_2}+P_{0,n_1}P_{0,n_2}P_{1,n_2}} \tag{14}$$

The result can also be checked against the special case for $n_1 = n_2 = n$. With $P_{0,n_1} = P_{0,n_2} = P_0$, and $P_{1,n_2} = P_1$, (14) reduces to,

$$\rho_a = \frac{P_0 + P_0P_1 + P_0^2P_1}{1 + P_0P_1}$$, which is the result

obtained from (1), page 8 of Reference 1 when $k_1 = 1$, and $k_2 = 3$.

Plan (3): 2,3; 0,1

State at ith trial

State at (i-1)st trial		00	01	10	RO	R
	00	P_{0,n_2}	P_{1,n_2}	.	.	$1-\sum_{i=0}^{1} P_{i,n_2}$
	01	.	.	P_{0,n_2}	.	$1-P_{0,n_2}$
	10	P_{0,n_2}	.	.	.	$1-P_{0,n_2}$
	RO	P_{0,n_1}	.	.	.	$1-P_{0,n_1}$
	R	.	.	.	P_{0,n_1}	$1-P_{0,n_1}$

Fig. 4. Markov Chain for ChSP(n_1,n_2)-0,1 Plan: (k_1,k_2; C_1,C_2 = 2,3; 0,1)

$$\rho_{00} = P_{0,n_2}\rho_{00} + P_{0,n_2}\rho_{10} + P_{0,n_1}\rho_{R0} \tag{15}$$

$$\rho_{01} = P_{1,n_2}\rho_{00} \tag{16}$$

$$\rho_{10} = P_{0,n_2}\rho_{01} \tag{17}$$

$$\rho_{R0} = P_{0,n_1}\rho_{R} \tag{18}$$

$$\rho_{00} + \rho_{01} + \rho_{10} + \rho_{R0} + \rho_{R} = 1 \tag{19}$$

Combining equations as before,

$$\begin{aligned}\rho_R = (1-P_{0,n_2}-P_{0,n_2}^2 P_{1,n_2}) / (&1-P_{0,n_2}-P_0{}^2{}_{,n_2}P_{1,n_2}+P_{0,n_1}-P_{0,n_1}P_{0,n_2} \\ &-P_{0,n_1}P_0{}^2{}_{,n_2}P_{1,n_2}+P_0{}^2{}_{,n_1}+P_0{}^2{}_{,n_1}P_{1,n_2} \\ &+P_0{}^2{}_{,n_1}P_{0,n_2}P_{1,n_2})\end{aligned} \tag{20}$$

$$\rho a = 1-P_R = (P_{0,n_1} - P_{0,n_1}P_{0,n_2} - P_{0,n_1}P_{0,n_2}^{2}P_{1,n_2} + P_{0,n_1}^{2} + P_{0,n_1}^{2}P_{1,n_2}$$
$$+ P_{0,n_1}^{2}P_{0,n_2}P_{1,n_2}) \,/\, (1 - P_{0,n_2} - P_{0,n_2}^{2}P_{1,n_2} + P_{0,n_1}$$
$$- P_{0,n_1}P_{0,n_2} - P_{0,n_1}P_{0,n_2}^{2}P_{1,n_2} + P_{0,n_1}^{2} + P_{0,n_1}^{2}P_{1,n_2} + P_{0,n_1}^{2}P_{1,n_2}$$
$$+ P_{0,n_1}^{2}P_{0,n_2}P_{1,n_2}) \qquad (21)$$

Again letting $P_{0,n_1} = P_{0,n_2} = P_0$, and $P_{1,n_2} = P_1$, (21) reduces to,

$\rho a = P_0 + P_0^{2}P_1$, which is the result for the associated ChSP-0,1 plan with $n_1 = n_2 = n$.

Plan (4): 1,2; 0,2

State at (i-1)st trial		State at ith trial: 0	1	2	R
	0	P_{0,n_2}	P_{1,n_2}	P_{2,n_2}	$1 - \sum_{i=0}^{2} P_{i,n_2}$
	1	P_{0,n_2}	P_{1,n_2}	•	$1 - \sum_{i=0}^{1} P_{i,n_2}$
	2	P_{0,n_2}	•	•	$1 - P_{0,n_2}$
	R	P_{0,n_1}	•	•	$1 - P_{0,n_1}$

Fig. 5. Markov Chain for ChSP(n_1,n_2)-0,2 Plan: (k_1,k_2; C_1,C_2 = 1,2; 0,2)

$$\rho_0 = P_{0,n_2}\rho_0 + P_{0,n_2}\rho_1 + P_{0,n_2}\rho_2 + P_{0,n_1}\rho_R \qquad (22)$$

$$\rho_1 = P_{1,n_2}\rho_0 + P_{1,n_2}\rho_1 \qquad (23)$$

$$\rho_2 = P_{2,n_2}\rho_0 \tag{24}$$

$$\rho_0 + \rho_1 + \rho_2 + \rho_R = 1 \tag{25}$$

These combine to give,

$$\rho a = (P_{0,n_1} + P_{0,n_1} P_{2,n_2} - P_{0,n_1} P_{1,n_2} P_{2,n_2}) / (1 + P_{0,n_1} + P_{0,n_1} P_{2,n_2} - P_{0,n_1} P_{1,n_2} P_{2,n_2} - P_{0,n_2} - P_{1,n_2} - P_{0,n_2} P_{2,n_2} + P_{0,n_2} P_{1,n_2} P_{2,n_2}) \tag{26}$$

With $P_{0,n_1} = P_{0,n_2} = P_0$, $P_{1,n_2} = P_1$ and $P_{2,n_2} = P_2$, (26) reduces to,

$$\rho a = \frac{P_0 + P_0P_2 - P_0P_1P_2}{1 - P_1}$$, which agrees with (19), page 13, Reference 3.

<u>Plan (5): 1,3; 0,2</u>

State at ith trial

State at (i-1)st trial	00	01	10	11	02	20	R0	R
00	P_{0,n_2}	P_{1,n_2}	·	·	P_{2,n_2}	·	·	$1 - \sum_{i=0}^{2} P_{i,n_2}$
01	·	·	P_{0,n_2}	P_{1,n_2}	·	·	·	$1 - \sum_{i=0}^{1} P_{i,n_2}$
10	P_{0,n_2}	P_{1,n_2}	·	·	·	·	·	$1 - \sum_{i=0}^{1} P_{i,n_2}$
11	·	·	P_{0,n_2}	·	·	·	·	$1 - P_{0,n_2}$
02	·	·	·	·	·	P_{0,n_2}		$1 - P_{0,n_2}$
20	P_{0,n_2}	·	·	·	·	·	·	$1 - P_{0,n_2}$
R0	P_{0,n_2}	P_{1,n_2}	·	·	P_{2,n_2}	·	·	$1 - \sum_{i=0}^{2} P_{i,n_2}$
R	·	·	·	·	·	·	P_{0,n_1}	$1 - P_{0,n_1}$

Fig. 6. Markov Chain for ChSP(n_1,n_2)-0,2 Plan: (k_1,k_2; C_1,C_2 = 1,3; 0,2)

Plan (6): 2,3; 1,2

State at ith trial

State at (i-1)st trial	00	01	10	11	02	20	R0	R1	R
00	P_{0,n_2}	P_{1,n_2}	.	.	P_{2,n_2}	.	.	.	$1 - \sum_{i=0}^{2} P_{i,n_2}$
01	.	.	P_{0,n_2}	P_{1,n_2}	.	.	.	.	$1 - \sum_{i=0}^{1} P_{i,n_2}$
10	P_{0,n_2}	P_{1,n_2}	.	.	.	.	.	.	$1 - \sum_{i=0}^{1} P_{i,n_2}$
11	.	.	P_{0,n_2}	.	.	.	.	.	$1 - P_{0,n_2}$
02	.	.	.	.	.	P_{0,n_2}	.	.	$1 - P_{0,n_2}$
20	P_{0,n_2}	.	.	.	.	.	.	.	$1 - P_{0,n_2}$
R0	P_{0,n_1}	P_{1,n_1}	.	.	.	.	.	.	$1 - \sum_{i=0}^{1} P_{i,n_1}$
R1	.	.	P_{0,n_1}	.	.	.	.	.	$1 - P_{0,n_1}$
R	.	.	.	.	.	.	P_{0,n_1}	P_{1,n_1}	$1 - \sum_{i=0}^{1} P_{i,n_1}$

Fig. 7. Markov Chain for ChSP(n_1,n_2)-1,2 Plan: (k_1,k_2; C_1,C_2 = 2,3; 1,2)

Plan (7): 1,5; 0,2

State at ith trial

State at (i-1)st trial	0000	0001	0010	0100	1000	1100	1010	1001	0110	0101	0011	0002	0020	0200	2000	R000	R001	R010	R011	R002	R020	R00	R01	R02	R0	R
0000	P_{0,n_2}	P_{1,n_2}	.	.	.	.	.	.	.	.	.	P_{2,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	.	$1-\sum_{i=0}^{2} P_{i,n_2}$
0001	.	.	P_{0,n_2}	.	.	.	.	.	.	.	P_{1,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$1-\sum_{i=0}^{1} P_{i,n_2}$
0010	.	.	.	P_{0,n_2}	.	.	.	.	.	P_{1,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$1-\sum_{i=0}^{1} P_{i,n_2}$
0100	.	.	.	.	P_{0,n_2}	.	.	P_{1,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$1-\sum_{i=0}^{1} P_{i,n_2}$
1000	P_{0,n_2}	P_{1,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$1-\sum_{i=0}^{1} P_{i,n_2}$
1100	.	.	.	.	P_{0,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$1-P_{0,n_2}$
1010	.	.	.	P_{0,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$1-P_{0,n_2}$
1001	.	.	P_{0,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$1-P_{0,n_2}$
0110	.	.	.	.	.	P_{0,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$1-P_{0,n_2}$
0101	.	.	.	.	.	.	P_{0,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$1-P_{0,n_2}$
0011	.	.	.	.	.	.	.	.	P_{0,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$1-P_{0,n_2}$
0002	.	.	.	.	.	.	.	.	.	.	.	.	P_{0,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	$1-P_{0,n_2}$
0020	.	.	.	.	.	.	.	.	.	.	.	.	.	P_{0,n_2}	.	.	.	.	.	.	.	.	.	.	.	$1-P_{0,n_2}$
0200	.	.	.	.	.	.	.	.	.	.	.	.	.	.	P_{0,n_2}	.	.	.	.	.	.	.	.	.	.	$1-P_{0,n_2}$
2000	P_{0,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$1-P_{0,n_2}$
R000	P_{0,n_2}	P_{1,n_2}	.	.	.	.	.	.	.	.	.	P_{2,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	.	$1-\sum_{i=0}^{2} P_{i,n_2}$
R001	.	.	P_{0,n_2}	.	.	.	.	.	.	.	P_{1,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$1-\sum_{i=0}^{1} P_{i,n_2}$
R010	.	.	.	P_{0,n_2}	.	.	.	.	.	P_{1,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$1-\sum_{i=0}^{1} P_{i,n_2}$
R011	.	.	.	.	.	.	.	.	P_{0,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	$1-P_{0,n_2}$
R002	.	.	.	.	.	.	.	.	.	.	.	.	P_{0,n_2}	.	.	.	.	.	.	.	.	.	.	.	.	$1-P_{0,n_2}$
R020	.	.	.	.	.	.	.	.	.	.	.	.	.	P_{0,n_2}	.	.	.	.	.	.	.	.	.	.	.	$1-P_{0,n_2}$
R00	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	P_{0,n_2}	P_{1,n_2}	.	.	P_{2,n_2}	.	.	.	.	.	$1-\sum_{i=0}^{2} P_{i,n_2}$
R01	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	P_{0,n_2}	P_{1,n_2}	.	.	.	.	.	.	$1-\sum_{i=0}^{1} P_{i,n_2}$
R02	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	P_{0,n_1}	.	.	.	.	$1-P_{0,n_2}$
R0	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	P_{0,n_2}	P_{1,n_2}	P_{2,n_2}	.	$1-\sum_{i=0}^{2} P_{i,n_2}$
R	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	P_{0,n_1}	$1-P_{0,n_1}$

Fig. 8. Markov Chain for ChSP(n_1,n_2)-0,2 Plan: (k_1,k_2; C_1,C_2 = 1,5; 0,2)

Plan (8): 1,2; 0,3

State at ith trial

State at (i-1)st trial		0	1	2	3	R
	0	P_{0,n_2}	P_{1,n_2}	P_{2,n_2}	P_{3,n_2}	$1-\sum_{i=0}^{3}P_{i,n_2}$
	1	P_{0,n_2}	P_{1,n_2}	P_{2,n_2}	.	$1-\sum_{i=0}^{2}P_{i,n_2}$
	2	P_{0,n_2}	P_{1,n_2}	.	.	$1-\sum_{i=0}^{1}P_{i,n_2}$
	3	P_{0,n_2}	.	.	.	$1-P_{0,n_2}$
	R	P_{0,n_1}	.	.	.	$1-P_{0,n_1}$

Fig. 9. Markov Chain for ChSP(n_1,n_2)-0,3 Plan: (k_1,k_2; C_1,C_2 = 1,2; 0,3)

$$\rho_0 = P_{0,n_2}\rho_0 + P_{0,n_2}\rho_1 + P_{0,n_2}\rho_2 + P_{0,n_2}\rho_3 + P_{0,n_1}\rho_R \tag{27}$$

$$\rho_1 = P_{1,n_2}\rho_0 + P_{1,n_2}\rho_1 + P_{1,n_2}\rho_2 \tag{28}$$

$$\rho_2 = P_{2,n_2}\rho_0 + P_{2,n_2}\rho_1 \tag{29}$$

$$\rho_3 = P_{3,n_2}\rho_0 \tag{30}$$

$$\rho_0 + \rho_1 + \rho_2 + \rho_3 + \rho_R = 1 \tag{31}$$

These combine to give,

$$\rho_a = [P_{0,n_1}(1+P_{2,n_2}+P_{3,n_2}-P_{1,n_2}P_{3,n_2}-P_{1,n_2}P_{2,n_2}P_{3,n_2})] \,/\, [1+P_{0,n_1}(1+P_{2,n_2}+P_{3,n_2}-P_{1,n_2}P_{3,n_2}-P_{1,n_2}P_{2,n_2}P_{3,n_2})-P_{0,n_2}(1+P_{2,n_2}+P_{3,n_2}-P_{1,n_2}P_{3,n_2}-P_{1,n_2}P_{2,n_2}P_{3,n_2})-P_{1,n_2}-P_{1,n_2}P_{2,n_2}] \tag{32}$$

With $P_{0,n_1} = P_{0,n_2} = P_0$, $P_{1,n_2} = P_1$, $P_{2,n_2} = P_2$, and $P_{3,n_2} = P_3$, (32) reduces to,

$$Pa = \frac{P_0+P_0P_2+P_0P_3-P_0P_1P_3-P_0P_1P_2P_3}{1-P_1-P_1P_2}$$ which

agrees with (9), page 4, Reference 4.

<u>Plan (9): 2,3; 0,3</u>

State at ith trial

State at (i-1)st trial	00	01	10	11	02	20	21	12	03	30	R0	R
00	P_{0,n_2}	P_{1,n_2}	·	·	P_{2,n_2}	·	·	·	P_{3,n_2}	·	·	$1-\sum_{i=0}^{3} P_{i,n_2}$
01	·	·	P_{0,n_2}	P_{1,n_2}	·	·	·	P_{2,n_2}	·	·	·	$1-\sum_{i=0}^{2} P_{i,n_2}$
10	P_{0,n_2}	P_{1,n_2}	·	·	P_{2,n_2}	·	·	·	·	·	·	$1-\sum_{i=0}^{2} P_{i,n_2}$
11	·	·	P_{0,n_2}	P_{1,n_2}	·	·	·	·	·	·	·	$1-\sum_{i=0}^{1} P_{i,n_2}$
02	·	·	·	·	·	P_{0,n_2}	P_{1,n_2}	·	·	·	·	$1-\sum_{i=0}^{1} P_{i,n_2}$
20	P_{0,n_2}	P_{1,n_2}	·	·	·	·	·	·	·	·	·	$1-\sum_{i=0}^{1} P_{i,n_2}$
21	·	·	P_{0,n_2}	·	·	·	·	·	·	·	·	$1-P_{0,n_2}$
12	·	·	·	·	·	P_{0,n_2}	·	·	·	·	·	$1-P_{0,n_2}$
03	·	·	·	·	·	·	·	·	·	P_{0,n_2}	·	$1-P_{0,n_2}$
30	P_{0,n_2}	·	·	·	·	·	·	·	·	·	·	$1-P_{0,n_2}$
R0	P_{0,n_1}	·	·	·	·	·	·	·	·	·	·	$1-P_{0,n_1}$
R	·	·	·	·	·	·	·	·	·	·	P_{0,n_1}	$1-P_{0,n_1}$

Fig. 10. Markov Chain for ChSP(n_1,n_2)-0,3 Plan: (k_1,k_2; C_1,C_2 = 2,3; 0,3)

By the use of the theory of Markov chains, the operating characteristics for a given value of fraction defective p over the effective range of p have been carried out for a large number of ChSP(n_1,n_2) plans. Plans with C_1,C_2 = 0,1; 0,2; 0,3 and 1,3; k_2 = 2,3 and 5 and $n_1 = 2n_2$, for n_2 = 5, 10, and 100 have been evaluated. OC curves for these plans are shown in Appendix A, Figs. 11 through 16.

Determination of ASN

As noted, the sampling plans being considered here result in a variable amount of sampling inspection. A measure of this amount as a function of the fraction defective is that of the average sample size or Average Sample Number (ASN) as it is called. This statistic can readily be determined for the n_1,n_2 chain sampling plans in terms of the limiting probabilities of the appropriate states.

In general the ASN is as follows:

$$ASN = n_1 P_{n_1} + n_2 P_{n_2}, \text{ where}$$

P_{n_1} = Proportion of trials (e.g. lots) for which n_1 is required.

P_{n_2} = Proportion of trials (e.g. lots) for which n_2 is required

$= 1-P_{n_1}$.

To illustrate with an example, consider the ChSP(n_1,n_2)-0,2 plan with parameters (n_1,n_2; 1,2; 0,2).

The possible states of the sampling process are: 0,1,2 and R, representing the cumulative defectives over k_2-1 trials. n_1 is required after each rejection for one (k_1=1) trial. This occurs with probability ρ_R.

Hence, $P_{n_1} = \rho_R$; $P_{n_2} = 1-\rho_R$, and

$$ASN = n_1\rho_R + n_2(1-\rho_R) = n_2 + n_1\rho_R - n_2\rho_R.$$

To explore this more generally a somewhat more complicated example must be considered, say ChSP(n_1,n_2)-0,2 with parameters $(n_1,n_2;\ 2,3,;\ 0,2)$.

The possible states are: 00, 01, 10, 11, 02, 20, R0, and R.

In this case we cannot say that n_1 will be required twice (k_1=2) after each rejection since rejections can occur consecutively.

However only two states can be reached following a rejection, i.e., R and R0, the 1st (i.e. R) when a second rejection results and the 2nd (i.e. R0) when an acceptance ($D \le C_1$) results. Thus n_1 is required after each rejection <u>and</u> after each acceptance just preceded by a rejection.

Hence $P_{n_1} = \rho_R + \rho_{R0}$

$$P_{n_2} = 1-\rho_R-\rho_{R0} = \sum_i \rho_i \text{ , } i = 00, 01, 10, 11, 02, 20$$

$ASN = n_1(\rho_R+\rho_{R0}) + n_2(1-\rho_R-\rho_{R0})$, which can be reduced as follows,

$$= n_1(\rho_R+P_0\rho_R) + n_2(1-\rho_R-P_0\rho_R)$$

$$= n_2 + n_1\,\rho_R(1+P_0) - n_2\rho_R(1+P_0)$$

where P_0 = probability of zero defectives in a sample of n_1 with fraction defective, p.

Note that for each of the two cases considered above, when $n_1 = n_2 = n$, ASN = n.

In general then,

for $k_1 = 0$, n_1 is not used.

$k_1 = 1$, n_1 is required after each rejection, for all k_2, i.e. $k_2 \ge k_1+1$

$$P_{n_1} = \rho_R$$

$k_1 \ge 2$, n_1 is required after each rejection and after from one to k_1-1 acceptances just preceded by a rejection.

$P_{n_1} = \sum_{j \epsilon \mathcal{S}} \rho_j$, i.e. the sum of the limiting probabilities of those states satisfying the above conditions. $\mathcal{S}$ corresponds to the set of all states of the associated Markov chain which involve an R and contain k_1 or less sample outcomes (counting R).
(Note that this holds for $k_1 = 1$ also, since the only state satisfying this condition is "R" itself)

$$\text{thus, ASN} = n_1 \sum_{j \epsilon \mathcal{S}} \rho_j + n_2 (1 - \sum_{j \epsilon \mathcal{S}} \rho_j) \tag{33}$$

A previous report[3] contains the procedure for solving the Markov chains for the limiting probabilities of the states with particular emphasis on the "R" state so that $\rho a = 1 - \rho_R$ is obtained. By the same procedure the limiting probabilities of the other states can be found in order to determine ASN. The states for which limiting probabilities are needed, as noted above, are those states containing an R. These are readily reduced to functions of ρ_R from the Markov chains by the following relation and by the nature of the construction of the states.

$$\rho_j = \sum_{i=1}^{s} p_{ij} \rho_i \quad , j \epsilon \mathcal{S} \tag{34}$$

where s = the total number of states

p_{ij} = the transition probabilities, generally denoted by P_i, i.e. the probability of getting i defectives in a sample of n with fraction defective, p.

e.g. for $(n_1,n_2;\ 2,3;\ 0,2)$

$\rho_{R0} = P_0\rho_R$, where P_0 = the probability of getting zero defectives in a sample of n_1 with fraction defective p.

and for $(n_1,\ n_2;\ 4,5;\ 1,2)$

$\rho_{R0001} = P_1\rho_{R00} = P_1P_0\rho_{R0} = P_1P_0{}^2\rho_R$, with P_0 and P_1 similary defined.

For the special case of $n_1 = 2n_2$ which applies to all of the specific evaluations carried out in the present report, the formulation of ASN reduces further. In this case expression (33) becomes,

$$ASN = n_2 + n_2 \sum_{j\epsilon S} \rho_j \tag{35}$$

Further study has shown that for the ChSP-0,1; 0,2; 1,2; 0,3 and 1,3 plans with k_2 = 2,3 and 5 which are considered in this report, five distinct formulas for ASN result. These are as follows:

For all plans with $k_1 = 1$, from (1,2; 0,1) to (1,5; 1,3),

$$ASN \doteq n_2 + n_2\rho_R, \tag{36}$$

For plans with C_1,C_2 = 0,1; 0,2 and 0,3 and $k_1 = 2$

$$ASN \doteq n_2 + n_2(1+P_0)\ \rho_R, \tag{37}$$

For plans with C_1,C_2 = 0,1; 0,2 and 0,3 and $k_1 = 4$,

$$ASN = n_2 + n_2(1+P_0 + P_0^2 + P_0^3)\ \rho_R\ , \tag{38}$$

For plans with $C_1, C_2 = 1,2$ and $1,3$ and $k_1 = 2$,

$$ASN = n_2 + n_2(1+P_0 + P_1)\, \rho_R \qquad (39)$$

For plans with $C_1, C_2 = 1,2$ and $1,3$ and $k_1 = 4$,

$$ASN = n_2 + n_2\, (1+P_0 + P_0^2 + P_0^3 + 2P_0P_1 + 3P_0^2P_1)\, \rho_R, \qquad (40)$$

where in all cases P_0 and P_1 denote the probabilities of 0 and 1 defectives in a sample of n_1 with fraction defective, p.

Formulas (36) through (40) have been used to obtain the results which are presented in Appendix B. A set of ASN curves for n_1=200, n_2=100 for k_1, k_2=1,2; 1,3; and 2,3 is shown in Fig. 17. Another set for k_1,k_2=1,5; 2,5; and 4,5 is shown in Fig. 18. These sets correspond directly to the ChSP plans for which the OC curves are presented in Figs. 15 and 16, Appendix A. The ASN curves for the sample size combination, n_1=200, n_2=100, are shown as being representative of the effect of the ChSP parameters on this property.

DISCUSSION OF OPERATING CHARACTERISTICS

The OC curves shown in Figs. 11 through 16 of Appendix A have been arranged differently than those of the preceding reports. Three sets of two pages each are used for the different sample size combinations--Figs. 11 and 12 for n_1=10, n_2=5; Figs. 13 and 14 for n_1=20, n_2=10; and Figs. 15 and 16 for n_1=200, n_2=100. Each chart within a page contains OC curves for different k_1,k_2 combinations--k_1,k_2=1,2; 1,3; and 2,3 on the first page of a set and k_1,k_2=1,5; 2,5; and 4,5 on the second page of a set. Thus the effect of changing k_1 is shown between each of the individual charts as is the effect of changing k_2 also. The effect of different C_i is shown within each individual chart. By this arrangement the combinations of C_i, i.e. C_1,C_2=0,1; 0,2 and 1,2; 0,3 and 1,3, of three preceding reports[1,3,4] are all covered in this single report for the more general ChSP(n_1,n_2)-C_1,C_2 plans. The OC curves are of Type B*, based on probabilities of sampling from a process.

By comparing the OC curves of this report with those of the previous reports it can be seen that all of the OC curves presented here are tightened considerably over their constant n counterparts (where $n=n_2$ of these plans); the scales have been halved. However, within this tightened range of p values it is seen that the effects of k_1,k_2,C_1, and C_2 are the same as for the constant n case. We need only to summarize these effects, then.

Effect of k_1 and k_2

Increasing k_1 from 1 up to k_2-1 has the effect of tightening (i.e. lowering) the OC curves, more drastically for ChSP(n_1,n_2)-0,C_2 than for ChSP(n_1,n_2)-1,C_2. (The k_1=0 plans are not shown due to the poor discrimination of these plans and because they are not part of the ChSP(n_1,n_2) set of plans (i.e. they involve only one stage). Increasing k_2 also has the effect of tightening the OC curves. However, in the case of a fixed k_1, say k_1=1, this tightening is accompanied by a somewhat poorer discrimination. For the plans in which $k_1=k_2-1$, the effect of increasing k_2 is to tighten the OC curves considerably.

Effect of C_1 and C_2

In the ChSP-C_1,C_2 plans of previous reports it was noted that the effect of using a 0,1; 0,2; or 0,3 chain sampling plan is to add a swelling on the

* See Reference 8, pp. 56-60.

underlying OC curve of the c=0, given n, single sampling plan. It was also noted that the swelling increases (particularly for low values of fraction defective, p) by going successively from a 0,1 to a 0,2 and 0,3 plan. It can be seen from the OC curves presented here that a similiar effect is true for the ChSP(n_1,n_2)-0,C_2 plans, although here the swelling is added to the c=0, n_1, single sampling plan. The ChSP(n_1,n_2)-1,C_2 plans, like their constant n counterparts, have less discriminating OC curves.

The cost that is incurred in terms of additional inspection effort for the greatly improved discrimination and tightening of the OC curves of the ChSP(n_1,n_2)-C_1,C_2 plans (with n_1=2n_2) is provided, to a large extent, by the average sample number, ASN. As noted, sample sets of ASN curves for n_1=200, n_2=100 are shown in Figs. 17 and 18 of Appendix B. It can be seen from these curves that ASN is effected most significantly by the parameter k_1 (which, of course, specifies the number of samples of n_1 required after each rejection). In general, the curves tend to support the use of plans with the smaller k_1 values, i.e. k_1=1 or 2.

To facilitate comparison of the ChSP($n_1 n_2$)-C_1,C_2 plans with their ChSP-C_1,C_2 counterparts, two additional sets of OC curves are presented in Figs. 19 and 20 of Appendix C. Fig. 19 contains OC curves for parameter sets; k_1,k_2=1,2; 1,3; 2,3, and 1,5; C_1,C_2=0,1 and 0,2 for n_1,n_2=20,10 and for n=10. Fig. 20 contains OC curves for parameter sets: k_1,k_2=1,2, 1,3, 2,3, and 2,5; C_1,C_2=0,2 and 0,3 for n_1,n_2=200,100 and for n=100. The curves clearly illustrate the effect of the larger sample size in the first stage, i.e. n_1=2n_2.

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Appendix A: Operating Characteristic Curves for ChSP(n_1,n_2)-C_1,C_2 Plans

OC curves for the following plans are presented here. Figs. 11, 12; 13, 14; and 15, 16 give OC curves for sample size combinations, n_1,n_2=10,5; 20,10; and 200,100 respectively.

Figs. 11.1, 13.1 and 15.1
n_1,n_2=10,5; 20,10; 200,100

k_1	k_2	C_1	C_2
1	2	1	3
1	2	1	2
1	2	0	3
1	2	0	2
1	2	0	1

Figs. 11.2, 13.2 and 15.2
n_1,n_2=10,5; 20,10; 200,100

k_1	k_2	C_1	C_2
1	3	1	3
1	3	1	2
1	3	0	3
1	3	0	2
1	3	0	1

Figs. 11.3, 13.3 and 15.3
n_1,n_2=10,5; 20,10; 200,100

k_1	k_2	C_1	C_2
2	3	1	3
2	3	1	2
2	3	0	3
2	3	0	2
2	3	0	1

Figs. 12.1, 14.1 and 16.1
n_1,n_2=10,5; 20,10; 200,100

k_1	k_2	C_1	C_2
1	5	1	3
1	5	1	2
1	5	0	3
1	5	0	2
1	5	0	1

Figs. 12.2, 14.2 and 16.2
n_1,n_2=10,5; 20,10; 200,100

k_1	k_2	C_1	C_2
2	5	1	3
2	5	1	2
2	5	0	3
2	5	0	2
2	5	0	1

Figs. 12.3, 14.3 and 16.3
n_1,n_2=10,5; 20,10; 200,100

k_1	k_2	C_1	C_2
4	5	1	3
4	5	1	2
4	5	0	3
4	5	0	2
4	5	0	1

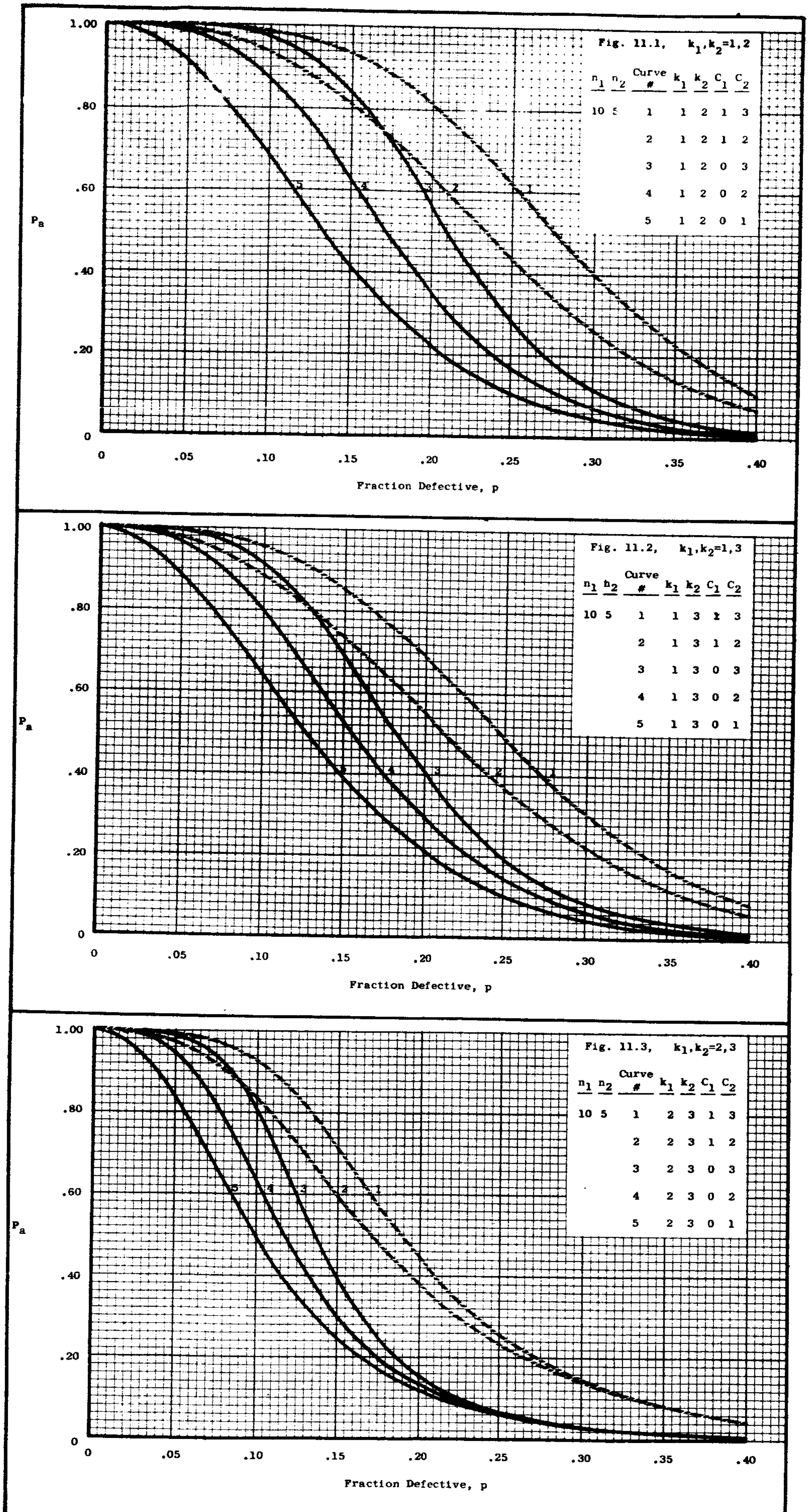

Fig. 11. OC Curves for ChSP(10,5)-C_1,C_2 Plans, k_1,k_2 = 1,2; 1,3; 2,3.

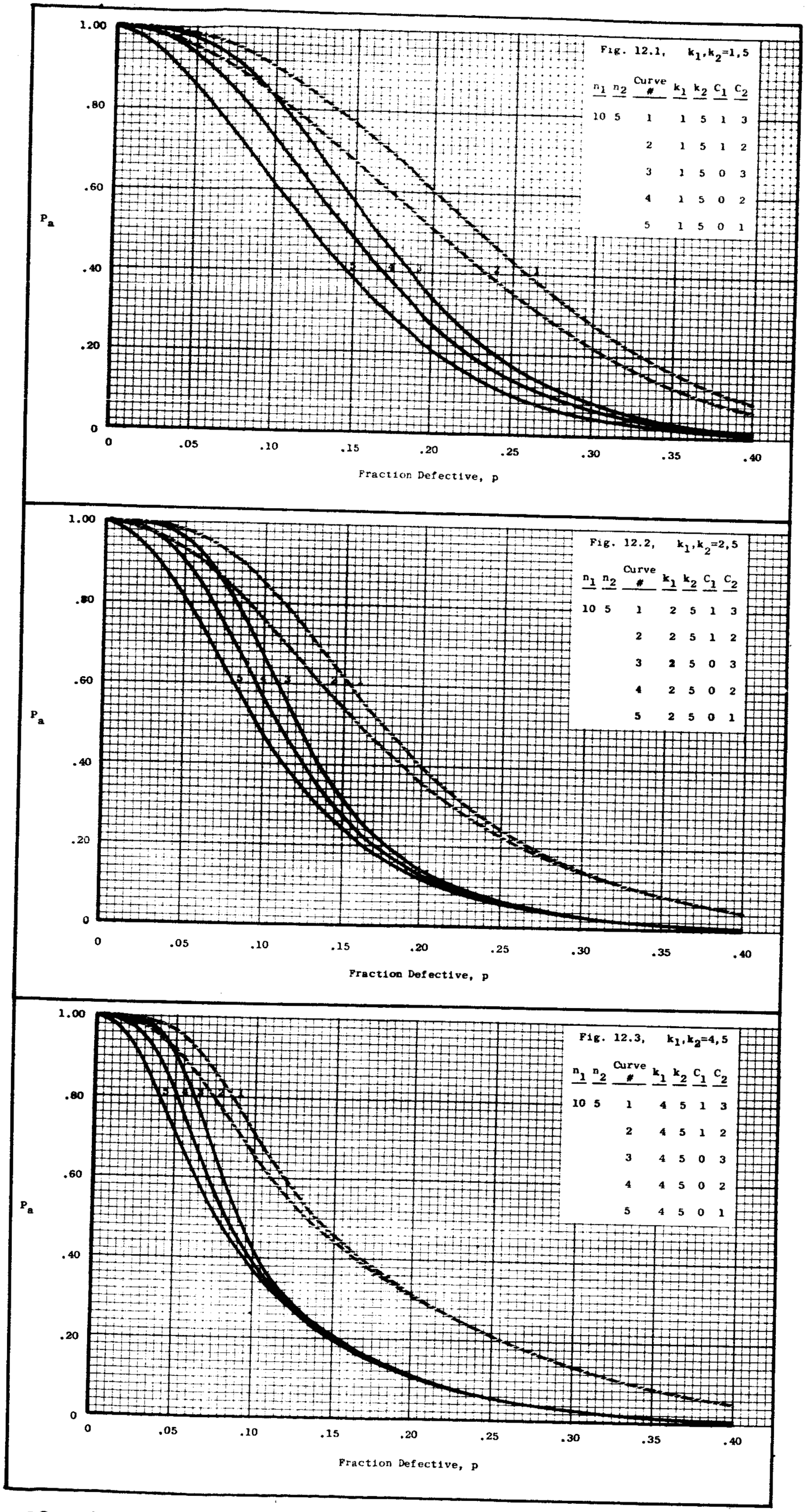

Fig. 12. OC Curves for ChSP(10,5)-C_1,C_2 Plans, k_1,k_2 = 1,5; 2,5; 4,5.

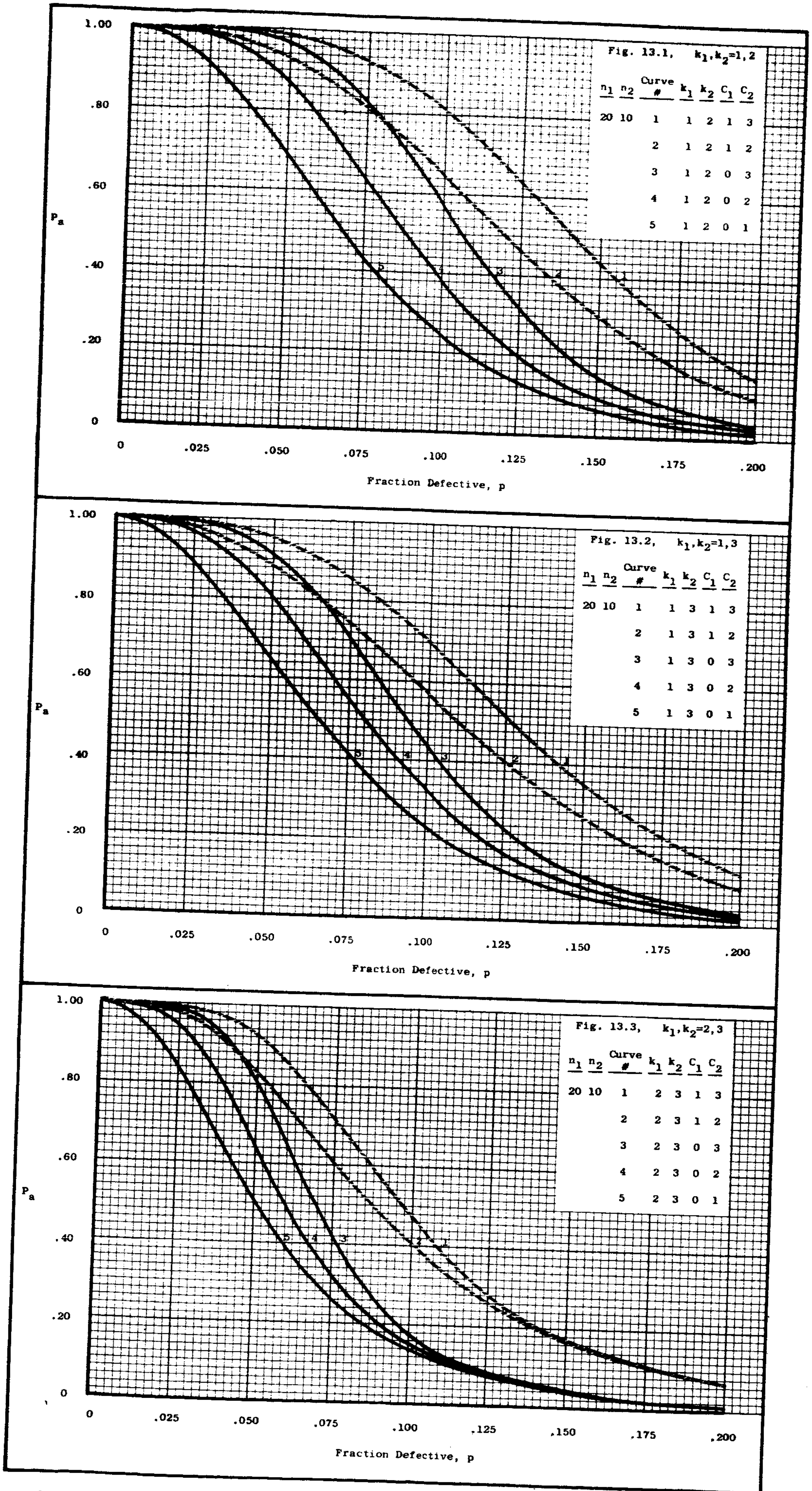

Fig. 13. OC Curves for ChSP(20,10)-C_1,C_2 Plans, k_1,k_2 = 1,2; 1,3; 2,3.

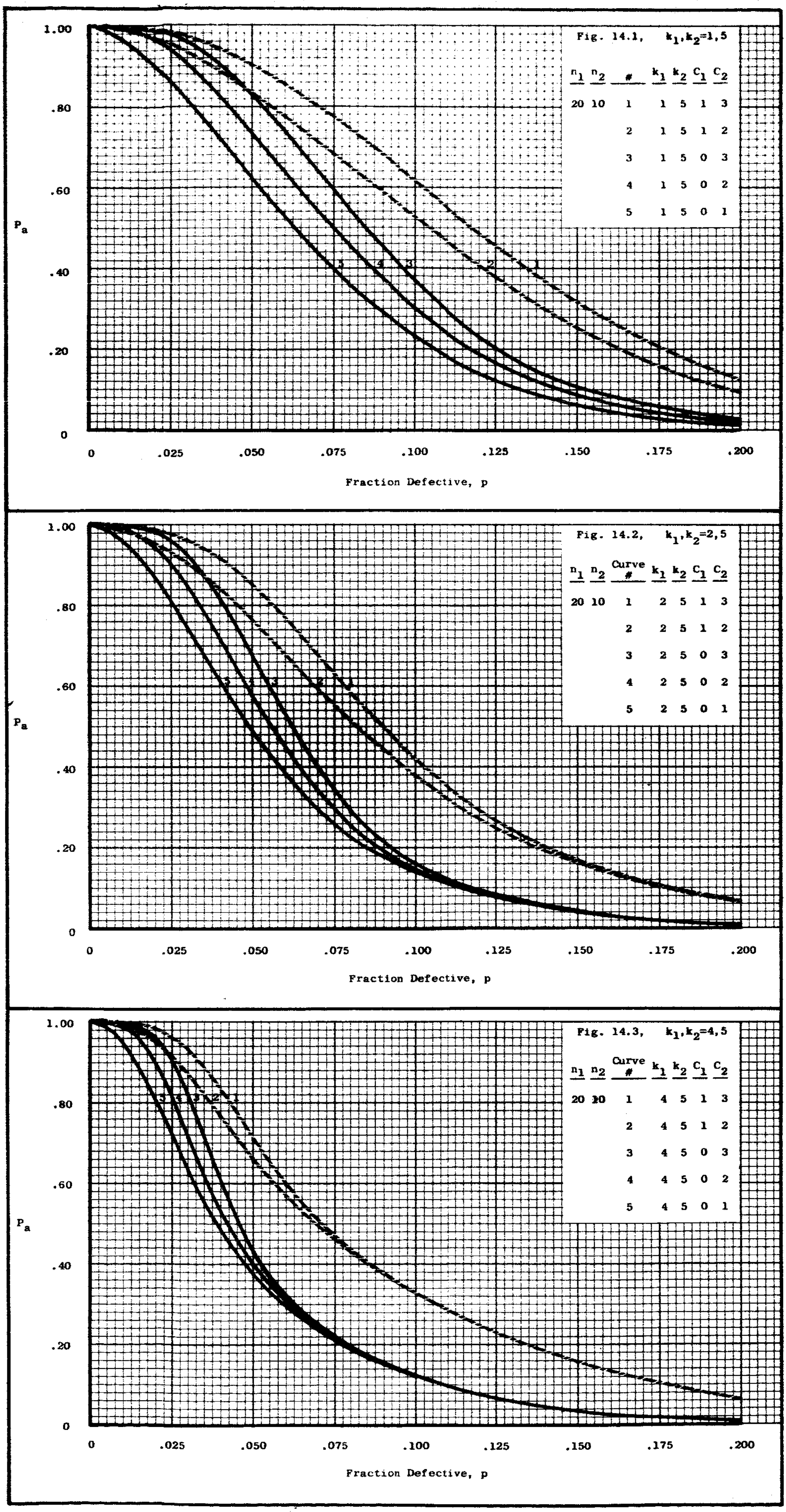

Fig. 14. OC Curves for ChSP(20,10)-C_1,C_2 Plans, k_1,k_2 = 1,5; 2,5; 4,5.

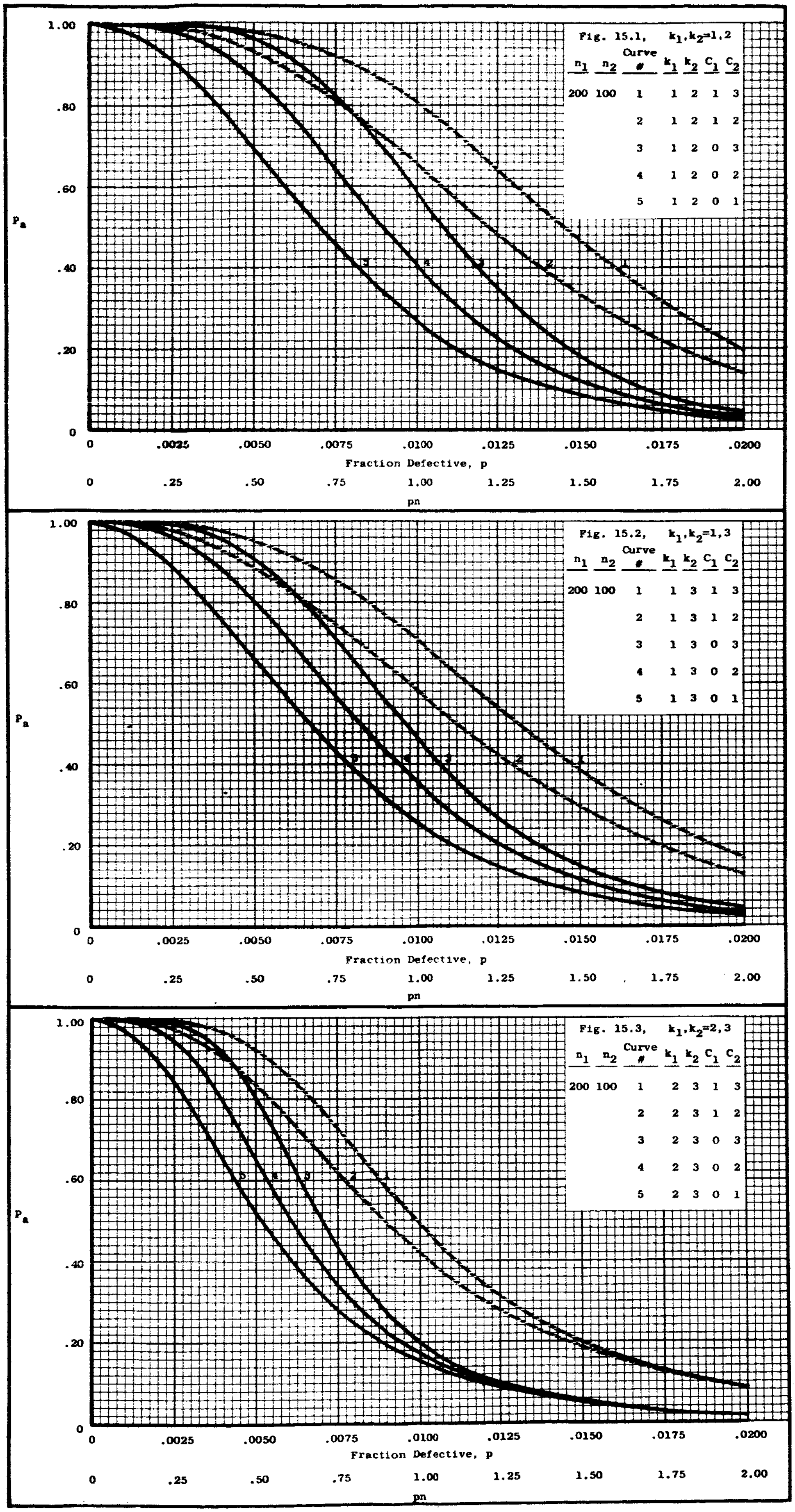

Fig. 15. OC Curves for ChSP(200,100)-C_1,C_2 Plans, k_1,k_2 = 1,2; 1,3; 2,3.

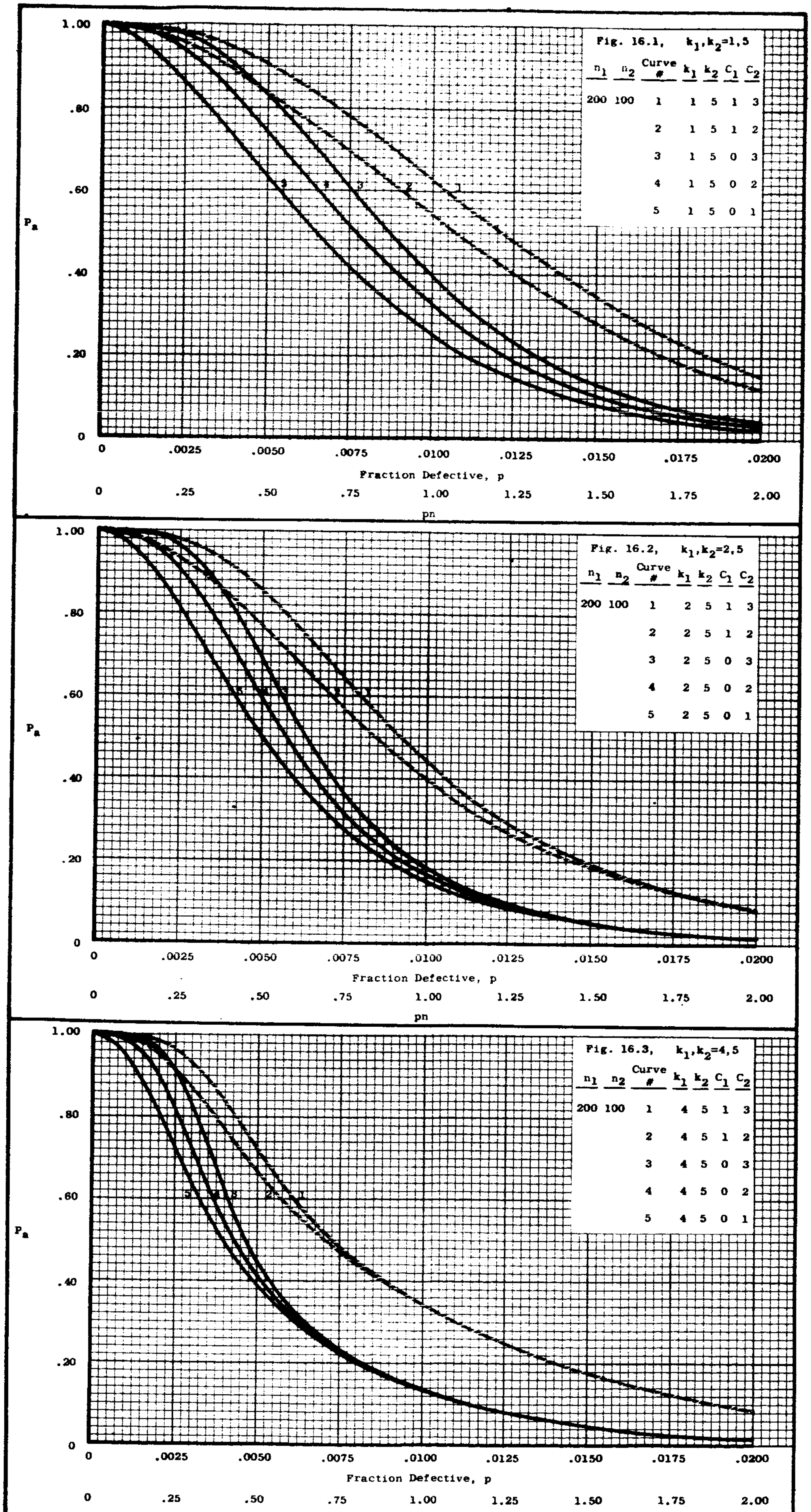

Fig. 16. OC Curves for ChSP(200,100)-C_1,C_2 Plans, k_1,k_2 = 1,5; 2,5; 4,5.

Appendix B: Average Sample Number Curves for ChSP(n_1,n_2)-C_1,C_2 Plans

ASN curves for the following plans are presented here, for the sample size combination n_1,n_2=200,100.

Fig. 17.1

k_1	k_2	C_1	C_2
1	2	1	3
1	2	1	2
1	2	0	3
1	2	0	2
1	2	0	1

Fig. 17.2

k_1	k_2	C_1	C_2
1	3	1	3
1	3	1	2
1	3	0	3
1	3	0	2
1	3	0	1

Fig. 17.3

k_1	k_2	C_1	C_2
2	3	1	3
2	3	1	2
2	3	0	3
2	3	0	2
2	3	0	1

Fig. 18.1

k_1	k_2	C_1	C_2
1	5	1	3
1	5	1	2
1	5	0	3
1	5	0	2
1	5	0	1

Fig. 18.2

k_1	k_2	C_1	C_2
2	5	1	3
2	5	1	2
2	5	0	3
2	5	0	2
2	5	0	1

Fig. 18.3

k_1	k_2	C_1	C_2
4	5	1	3
4	5	1	2
4	5	0	3
4	5	0	2
4	5	0	1

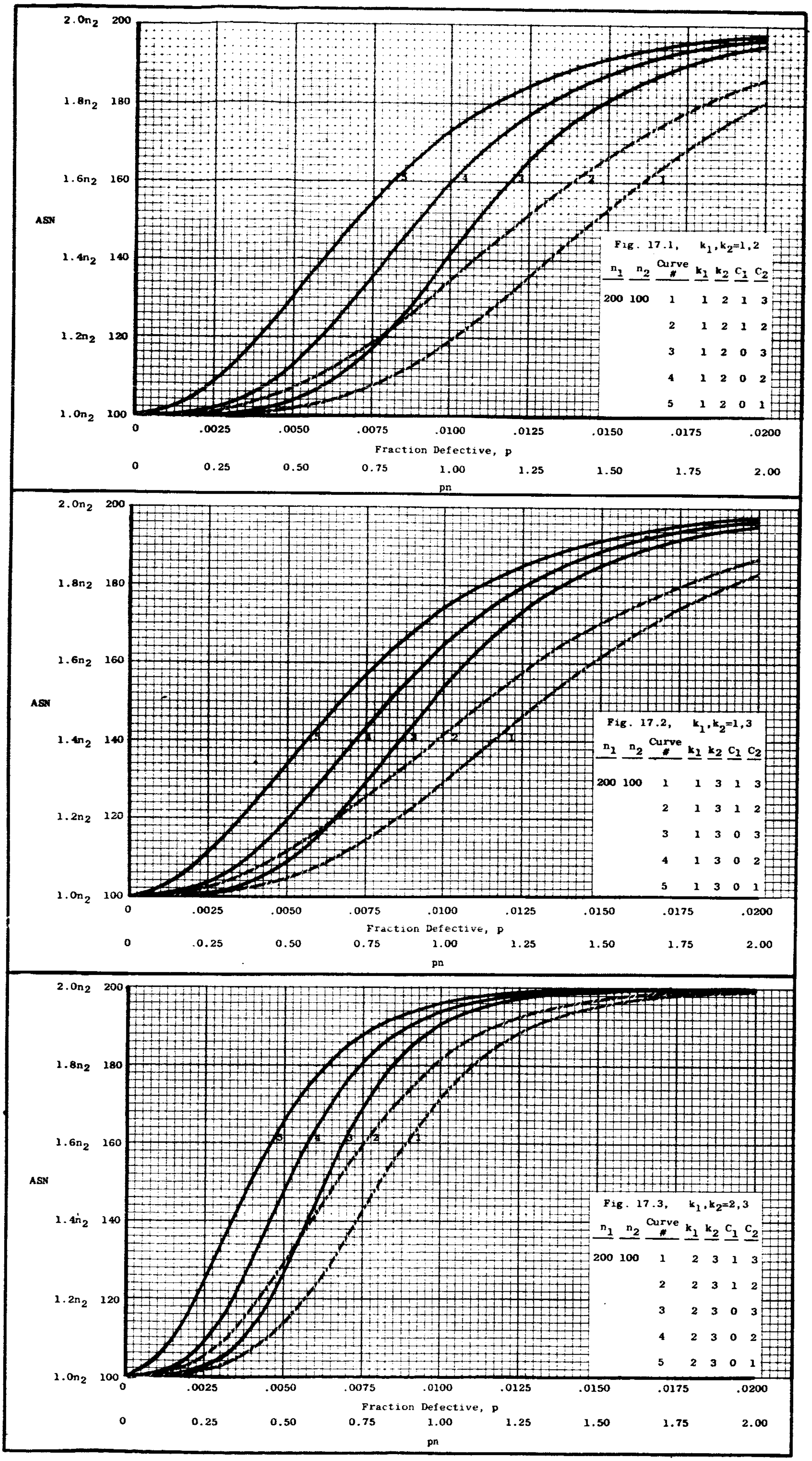

Fig. 17. ASN Curves for ChSP(200,100)-C_1,C_2 Plans, k_1,k_2 = 1,2; 1,3; 2,3.

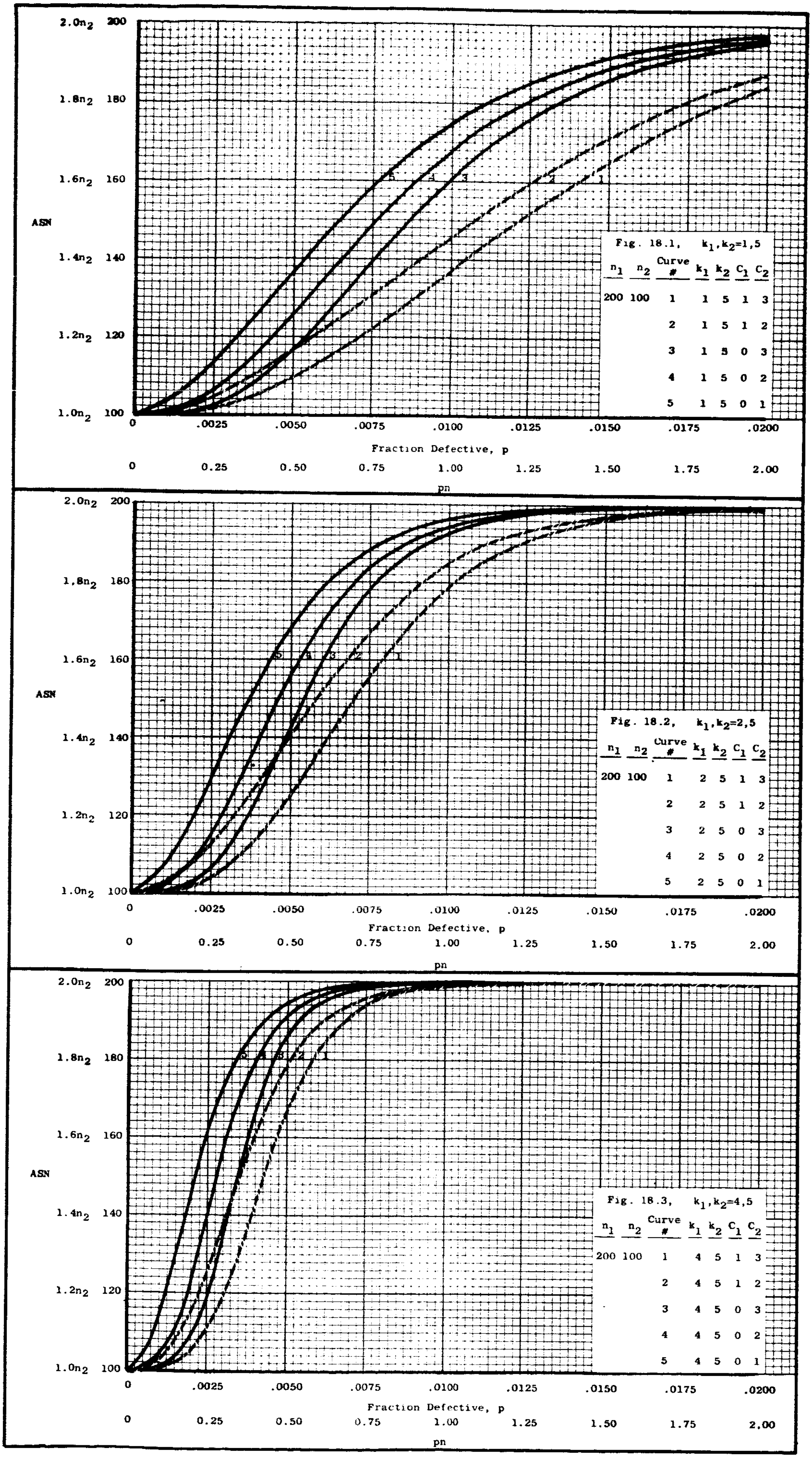

Fig. 18. ASN Curves for ChSP(200,100)-C_1,C_2 Plans, k_1,k_2 = 1,5; 2,5; 4,5.

Appendix C: Operating Characteristic Curves for ChSP(n_1,n_2)-C_1,C_2 and ChSP-C_1,C_2 Plans.

OC curves for the following plans are presented here.

Fig. 19.1

n_1	n_2	k_1	k_2	C_1	C_2
10	10	1	2	0	2
		1	2	0	1
20	10	1	2	0	2
		1	2	0	1

Fig. 19.2

n_1	n_2	k_1	k_2	C_1	C_2
10	10	1	3	0	2
		1	3	0	1
20	10	1	3	0	2
		1	3	0	1

Fig. 19.3

n_1	n_2	k_1	k_2	C_1	C_2
10	10	2	3	0	2
		2	3	0	1
20	10	2	3	0	2
		2	3	0	1

Fig. 19.4

n_1	n_2	k_1	k_2	C_1	C_2
10	10	1	5	0	2
		1	5	0	1
20	10	1	5	0	2
		1	5	0	1

Fig. 20.1

n_1	n_2	k_1	k_2	C_1	C_2
100	100	1	2	0	3
		1	2	0	2
200	100	1	2	0	3
		1	2	0	2

Fig. 20.2

n_1	n_2	k_1	k_2	C_1	C_2
100	100	1	3	0	3
		1	3	0	2
200	100	1	3	0	3
		1	3	0	2

Fig. 20.3

n_1	n_2	k_1	k_2	C_1	C_2
100	100	2	3	0	3
		2	3	0	2
200	100	2	3	0	3
		2	3	0	2

Fig. 20.4

n_1	n_2	k_1	k_2	C_1	C_2
100	100	2	5	0	3
		2	5	0	2
200	100	2	5	0	3
		2	5	0	2

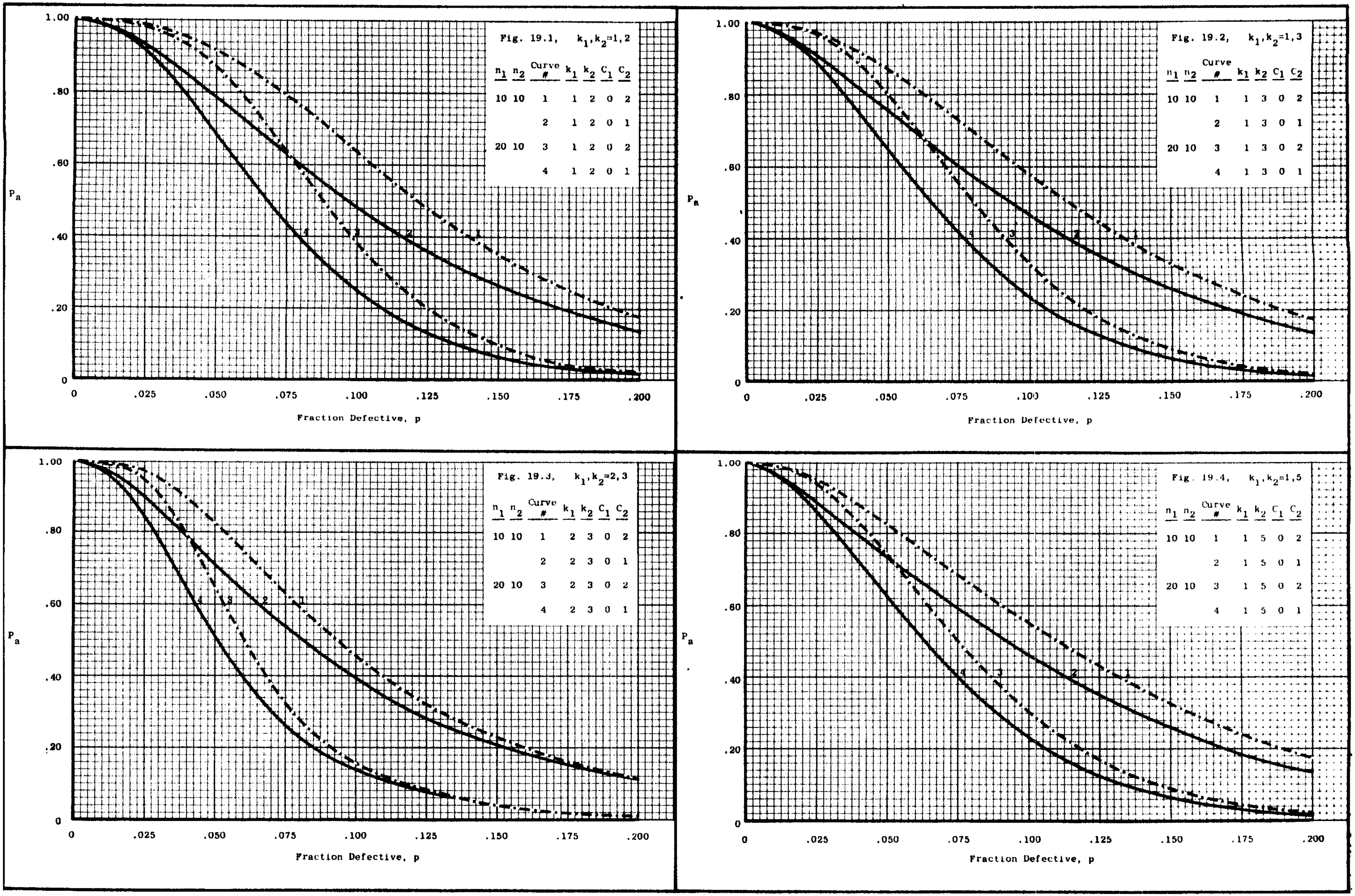

Fig. 19. Comparison of OC Curves for ChSP(20,10)-C_1,C_2 Plans and ChSP-C_1,C_2 Plans with n=10.

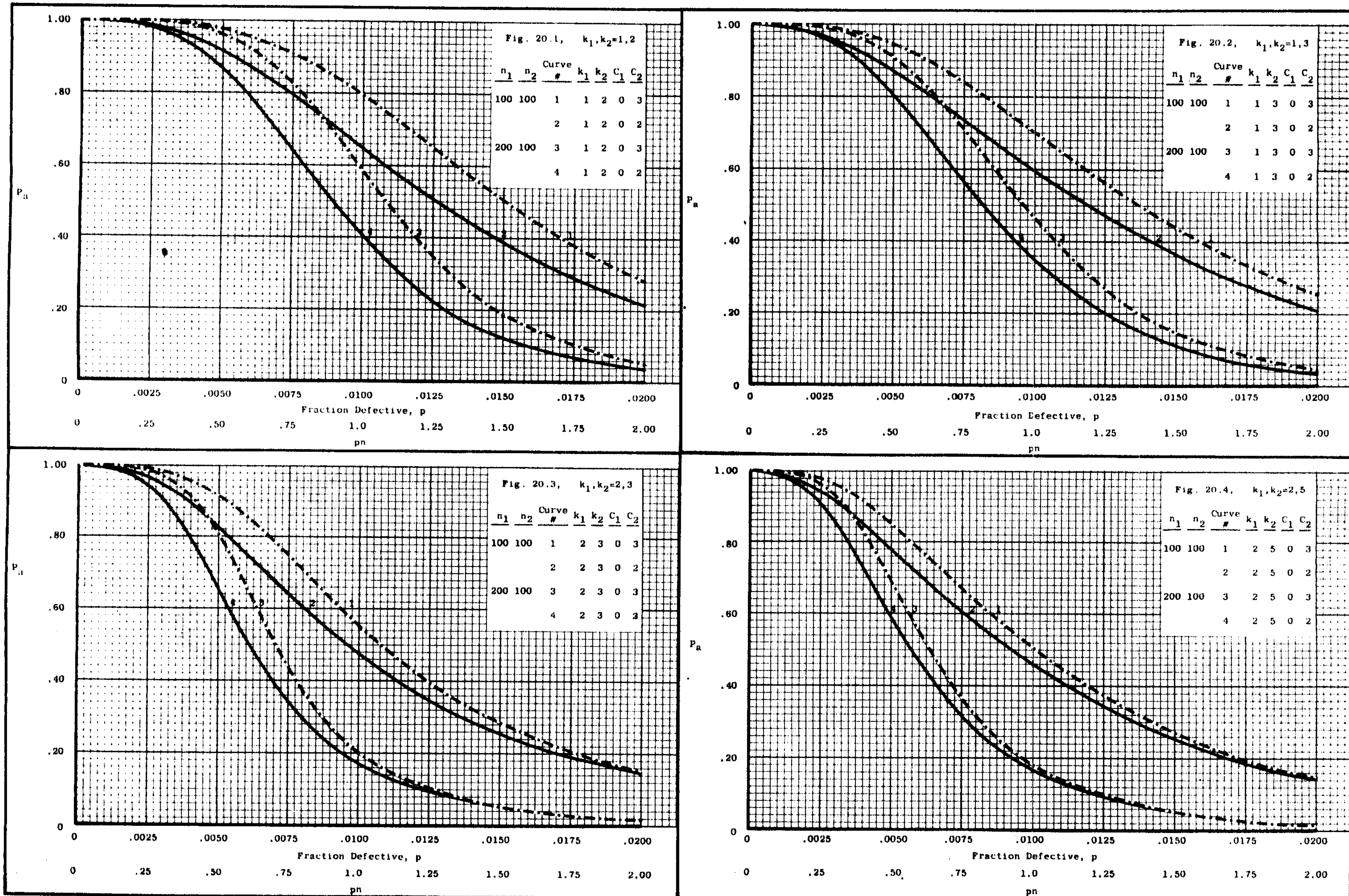

Fig. 20 Comparison of OC Curves for ChSP(200,100)-C_1,C_2 Plans and ChSP-C_1,C_2 Plans with n=100.

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13. ABSTRACT

This report presents a further generalization of the family of "two-stage" chain sampling inspection plans--the use of different sample sizes in the two stages. Plans presented in previous reports specify the use of the same sample size in both stages. Evaluation of the operating characteristics is accomplished by the Markov chain approach developed in the preceding reports. Markov chains for a number of plans are included and several algebraic solutions are developed. Since these plans involve a variable amount of sampling, an evaluation of the average sample number (ASN) is developed. OC curves are presented and discussed for plans with acceptance numbers, C_1,C_2 = 0,1; 0,2, 1,2; and 0,3, 1,3 and sample sizes, n_1,n_2 = 10,5; 20,10; and 200,100. ASN curves for the same plans for n_1,n_2 = 200,100 are also presented. Then to compare the two-sample-size plans with plans having only one sample size, OC curves for a number of each of these types of plans are presented. Comparisons indicate that improved discrimination is achieved by the two-sample-size plans.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Chain Sampling Sampling Inspection Plans Acceptance Sampling Cumulative-results Plans Markov Chains Operating Characteristics Average Sample Number						

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